A New Four-Port Automatic Network Analyzer: Part II—Theory

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Abstract—This paper presents a comprehensive theory of the new four-port network analyzer using only one power detector and one electronically adjustable reference load, (e.g., an adjustable short circuit or a varactor diode) described in a companion paper. A method based on annihilation operators for describing an n-port is introduced. This method is found convenient for analyzing different measurement situations. Hence, schemes for reflection measurements and transmission measurements are analyzed in detail.

For evaluation of the reflection (or transmission) coefficient, it is shown that seven (or five) parameters defining the system have to be determined. Calibration schemes for precision reflection and transmission measurements are described.

I. Introduction

IN A COMPANION paper [1], a new type of four-port network analyzer is described. The key components of the analyzer are, besides the four-port (e.g., a magic T), a variable reference load and only one power detector. The simple design and the moderate demands on component quality means that it should be ideal for millimeter-wave frequencies (compare [2]).

In this paper, schemes are described that can perform measurements of reflection or transmission coefficients.

A necessary condition for achieving accurate measurements with the new analyzer is knowledge of the so-called system parameters with a high degree of accuracy. By using a detector system with a large dynamic range, calibration procedures can be advised, which practically eliminate errors depending on irregularities in the reference load, as well as errors depending on the detector noise. The calibration is performed once and leads to measurement accuracies discussed in a companion paper [1].

Below, we will first derive relevant properties for an n-port system, and then focus on the particular systems to be used for reflection and transmission measurements, respectively. For these system descriptions, so-called generalized scattering parameters are used. A convenient way to obtain these parameters is suggested.

II. Wave Amplitude Theory for an N-Port

Referring to Fig. 1, we have for a general n-port

$$b_k = \sum_{l=1}^{n} S_{kl} \cdot a_l, \qquad k = 1, \cdots, n \tag{1}$$

where S_{kl} represents the internal scattering coefficients.

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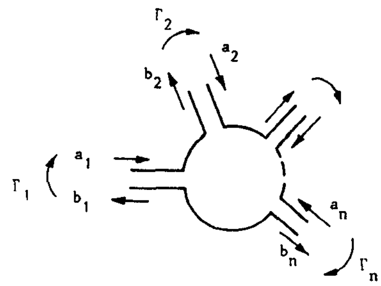

Fig. 1. Complex amplitudes of waves in an n-port.

Let e_q represent the incoming wave amplitude from the generator fed to port q. We then have

$$a_q = e_q + \Gamma_q \cdot b_q \tag{2a}$$

$$a_k = \Gamma_k \cdot b_k, \qquad \text{for } k \neq q. \tag{2b}$$

Substituting (2a) and (2b) into (1) gives

$$G \cdot X = H \tag{3a}$$

where

$$G = \begin{pmatrix} (S_{11}\Gamma_1 - 1) & S_{12} \cdot \Gamma_2 & \cdots & S_{1n} \cdot \Gamma_n \\ S_{21}\Gamma_1 & (S_{22} \cdot \Gamma_2 - 1) & \cdots & S_{2n} \cdot \Gamma_n \\ \vdots & & & \vdots \\ S_{n1}\Gamma_1 & & & (S_{nn}\Gamma_n - 1) \end{pmatrix}$$

$$X = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \quad H = \begin{pmatrix} -S_{1q} \\ -S_{2q} \\ \vdots \\ -S_{nq} \end{pmatrix} \cdot e_q. \tag{3b}$$

Equation (3a) has the solution

$$X = G^{-1} \cdot H. \tag{4}$$

From (4), the outgoing wave amplitude b_r in an arbitrary port number r can be obtained by applying Cramer's rule

$$b_r = e_q^* \frac{[F]}{[G]}. \tag{5}$$

The elements of the determinant $[G]$ in (5) is the same as the elements of the matrix in (3b), which means

$$G_{kl} = S_{kl} \cdot \Gamma_l, \qquad \text{for } l \neq k$$
$$G_{kl} = S_{kk} \cdot \Gamma_k - 1, \qquad \text{for } l = k.$$

The elements of the determinant $[F]$ in (5) are obtained by replacing the rth column in $[G]$ with the elements of the vector H in (3b), which means

$$
\begin{aligned}
F_{kl} &= S_{kl}\Gamma_l, && \text{for } l \neq r \text{ and } l \neq k \\
F_{kl} &= S_{kk}\Gamma_k - 1, && \text{for } l \neq r \text{ and } l = k \\
F_{kl} &= -S_{kq}, && \text{for } l = r.
\end{aligned}
\tag{6}
$$

A convenient method to expand (5) is suggested below. Thus, a so-called generalized scattering parameter (${}^{n}T_{rq}$) involving new entities ${}^{n}P_{rq}$ and Q_n defined below is introduced (n = number of ports of the system), viz.,

$$\frac{b_r}{e_q} = {}^{n}T_{rq} = \frac{{}^{n}P_{rq}}{Q_n} \tag{7a}$$

where

$${}^{n}P_{rq} = (-1)^{r+q+1} \cdot \left[S_{qr}^{*} \cdot \Gamma_q^{*} \cdot \Gamma_r^{*} \cdot \prod_{\substack{k=1 \\ k \neq q,r}}^{n} \left(1 + t \cdot S_{kk}^{*}\Gamma_k^{*}\right) \right] \cdot Z_n, \quad \text{for } r \neq q \tag{7b}$$

$$t = +1 \begin{cases} \text{if } k < r \text{ and } k > q \text{ or} \\ \text{if } k < q \text{ and } k > r \end{cases}$$

$$t = -1 \begin{cases} \text{if } k < r \text{ and } k < q \text{ or} \\ \text{if } k > r \text{ and } k > q \end{cases} \tag{7c}$$

$${}^{n}P_{qq} = -\left[\Gamma_q^{*} \cdot \prod_{\substack{k=1 \\ k \neq q}}^{n} \left(1 - S_{kk}^{*}\Gamma_k^{*}\right) \right] \cdot Z_n \tag{7}$$

$$Q_n = \left[\prod_{k=1}^{n} \left(1 - S_{kk}^{*}\Gamma_k^{*}\right) \right] \cdot Z_n, \quad \text{for all } r, q \text{ combinations} \tag{7d}$$

$$Z_n = \begin{vmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & & \vdots \\ S_{n1} & \cdots & S_{nn} \end{vmatrix} \cdot \prod_{k=1}^{n} \Gamma_k. \tag{7e}$$

S_{kl}^{*} and Γ_k^{*} are operators with annihilating functions. These are defined according to

$$\Gamma_k^{*} \cdot (\Gamma_1 \cdots \Gamma_{k-1} \cdot \Gamma_k \cdot \Gamma_{k+1} \cdots \Gamma_n) = (\Gamma_1 \cdots \Gamma_{k-1} \cdot \Gamma_{k+1} \cdots \Gamma_n) \tag{8a}$$

$$S_{kl}^{*} \cdot \begin{vmatrix} S_{11} & & \cdots & S_{1l} & \cdots & & S_{1n} \\ \vdots & & & \vdots & & & \vdots \\ & & S_{k-1l-1} & S_{k-1l} & S_{k-1l+1} & & \\ S_{k1} & \cdots & S_{kl-1} & S_{kl} & S_{kl+1} & \cdots & S_{kn} \\ \vdots & & S_{k+1l-1} & S_{k+1l} & S_{k+1l+1} & & \vdots \\ S_{n1} & & & S_{nl} & & & S_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} S_{11} & & \cdots & \cdots & & S_{1n} \\ \vdots & \cdots & S_{k-1,l-1} & S_{k-1,l+1} & \cdots & \vdots \\ \vdots & \cdots & S_{k+1,l-1} & S_{k+1,l+1} & \cdots & \vdots \\ S_{n1} & & & & & S_{nn} \end{vmatrix} \tag{8b}$$

which means that the complexity of (7e) is reduced when the operators S_{kl}^{*} and Γ_k^{*} are applied. The parameter ${}^{n}T_{rq}$ is a compact exact expression for the internal multiple scatterings in an arbitrary n-port for a wave fed to port q, arriving into port r. Fig. 4 gives a graphic illustration of ${}^{n}T_{rq}$ for the case $n = 4$, $q = 1, 3$. The parameters ${}^{n}T_{rq}$ defined by (7a–e) are very useful in the analysis of various systems with variable internal scattering coefficients.

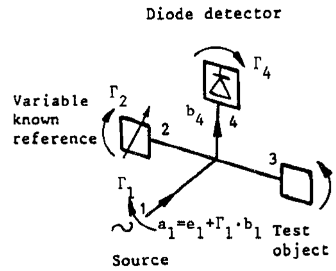

Fig. 2. Measurement setup for reflection measurements.

III. Reflection Measurements Using a Four-Port

In a reflectometer measurement setup, a four-port should be used as indicated in Fig. 2. Using (7) with $n = 4$, we get

$$b_4 = e_1 \cdot {}^{4}T_{41} = e_1 \cdot \frac{\left[S_{14}^{*} \cdot \Gamma_1^{*} \cdot \Gamma_4^{*} \cdot \left(1 + S_{22}^{*} \cdot \Gamma_2^{*}\right) \cdot \left(1 + S_{33}^{*} \cdot \Gamma_3^{*}\right)\right] \cdot Z_4}{\left(1 - S_{11}^{*}\Gamma_1^{*}\right)\left(1 - S_{22}^{*}\Gamma_2^{*}\right)\left(1 - S_{33}^{*}\Gamma_3^{*}\right)\left(1 - S_{44}^{*}\Gamma_4^{*}\right) Z_4} \tag{9}$$

where Z_4 is defined according to (7e).

Using the properties of the annihilation operators, we may write ${}^{4}T_{rq}$ in the form

$${}^{4}T_{41} = \frac{{}^{4}P_{41}}{Q_4} = \frac{\sum_{\mu=1}^{4} {}^{1}D_{\mu}}{\sum_{\mu=1}^{16} D_{\mu}} \tag{10a}$$

where ${}^{1}D_{\mu}$ and D_{μ} are determinants containing elements S_{kl} of order 1 to 3 and 1 to 4, respectively, multiplied by external reflection coefficients Γ_k.

From (9), the four terms of the numerator can be derived as

$${}^{1}D_1 = \begin{vmatrix} S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \\ S_{41} & S_{42} & S_{43} \end{vmatrix} \cdot \Gamma_2\Gamma_3 \quad {}^{1}D_2 = \begin{vmatrix} S_{21} & S_{22} \\ S_{41} & S_{42} \end{vmatrix} \cdot \Gamma_2$$

$${}^{1}D_3 = \begin{vmatrix} S_{31} & S_{33} \\ S_{41} & S_{43} \end{vmatrix} \cdot \Gamma_3 \quad {}^{1}D_4 = S_{41}. \tag{10b}$$

When the total expression in (10) is expanded, all terms representing unphysical scatterings (for example, $S_{41} \cdot S_{22} \cdot \Gamma_2$ in ${}^{1}D_2$) are cancelled. The expanded expression of (10) represents the infinite series of multiple scatterings obtained by following the scattering arrows in Fig. 4 if e_3 is put equal to zero.

For a fixed frequency, the only nonconstant parameters of Fig. 2 are Γ_2 and Γ_3, representing the reflection coefficients of the adjustable reference object and of the test object, respectively. We obtain an expression for the wave

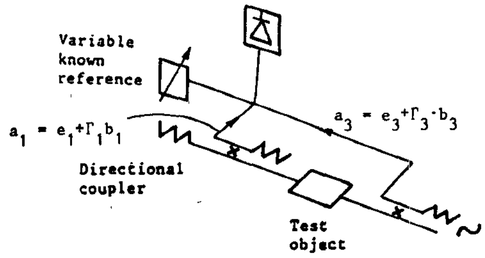

Fig. 3. Wave amplitudes in a mismatched system.

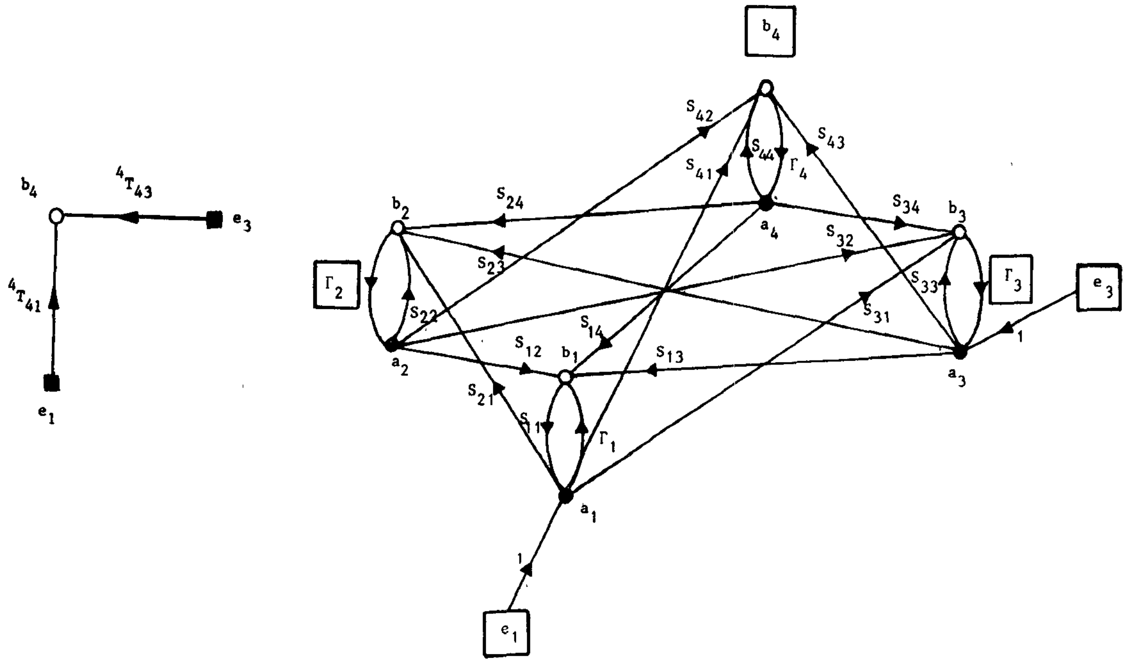

Fig. 4. The relation between the parameter $^4T_{41}$, $^4T_{43}$ and their original expressions in S_{kl}, Γ_k coefficients.

amplitude towards the detector accordingly

$$b_4 = \frac{K_1\Gamma_2\Gamma_3 + K_2\Gamma_2 + K_3\Gamma_3 + K_4}{K_5\Gamma_2\Gamma_3 + K_6\Gamma_2 + K_7\Gamma_3 + 1}. \tag{11a}$$

K_k in (11) consequently are frequency-dependent parameters defining all properties of the measurement setup, and can be expressed in terms if e_1, S_{kl}, Γ_1, and Γ_4.

Equation (10) written in the form of (11a) leads to

$$K_1\Gamma_2\Gamma_3 = \frac{e_1}{K}\cdot {}^1D_1$$

$$K_2\Gamma_2 = \frac{e_1}{K}\cdot {}^1D_2$$

$$K_3\Gamma_3 = \frac{e_1}{K}\cdot {}^1D_3 \quad K_4 = \frac{e_1}{K}\cdot {}^1D_4 \tag{11b}$$

where

$$K = 1 - S_{11}\Gamma_1 - S_{44}\Gamma_4 + \begin{vmatrix} S_{11} & S_{14} \\ S_{41} & S_{44} \end{vmatrix}\Gamma_1\Gamma_4.$$

In a fairly symmetric T, $S_{41} \approx 0$, $S_{42} \approx -S_{43}$, $S_{12} \approx S_{13}$, ($S_{ij} = S_{ji}$) yield. From (11b), we thus conclude that, in such a magic T, K_1 and K_4 must be small compared with K_2 and K_3.

For more details (see [3]), (11) is basic for the reflection measurement method described in the companion paper [1]. Knowledge about the approximate size of the parameters K_k makes it easier to select convenient calibration procedures for determining the final values of the complex parameters K_k.

IV. Transmission Measurements

A system for transmission measurements is shown in Fig. 3. The wave amplitude b_4 in the detector arm can be expressed as a function of two incoming wave amplitudes $e_1 = a_1 - b_1\Gamma_1$ and $e_3 = a_3 - b_3\Gamma_3$ in arms 1 and 3, respectively.

In the arrangement of Fig. 3, the wave amplitude b_4 can be written

$$b_4 = {}^4T_{41}\cdot e_1 + {}^4T_{43}\cdot e_3. \tag{12}$$

A graphic picture of (12) is shown in Fig. 4.

The factor $^4T_{41}$ can be obtained from (9). However, in the setup of Fig. 3, Γ_3 is a small and (for a fixed frequency) constant parameter contrary to the situation in Fig. 2. For a fixed frequency, the term $^4T_{41}$ thus contains only one variable, the reflection coefficient of the reference term Γ_2.

From (7), one concludes that $^4T_{41}$ and $^4T_{43}$ have the same denominator, while the four terms of the numerator of ${}^4T_{43}$ are different from those of ${}^4T_{41}$, viz.,

$$\begin{aligned} {}^4T_{43} &= \frac{{}^4P_{43}}{Q_4} \\ &= \frac{\left[S_{34}^*\Gamma_3^*\cdot\Gamma_4^*\cdot(1-S_{11}^*\Gamma_1^*)\cdot(1-S_{22}^*\Gamma_2^*)\right]\cdot Z_4}{Q_4} \\ &= \frac{\sum_{\mu=1}^{4} {}^3D_\mu}{\sum_{\mu=1}^{16} D_\mu} \end{aligned} \tag{13}$$

where Z_4 is defined according to (7e). From (7) and (8), the four ${}^3D_\mu$ terms can easily be derived.

The wave amplitudes e_1 and e_3 are related to each and other as

$$e_1 = \eta\cdot T\cdot e_3 \tag{14}$$

where T is the unknown transmission coefficient of the test object. The factor η describes the phase and amplitude difference between the input signals e_1 and e_3 caused by differences in the scattering properties of the two directional couplers. To minimize the multiple scatterings in the directional coupler, which links the test object and the four-port, the coupling factor should be -20 dB or less.

Equation (14) inserted in (12) leads to

$$b_4 = \frac{L_1\cdot T\cdot\Gamma_2 + L_2\cdot T + L_3\cdot\Gamma_2 + L_4}{L_5\cdot\Gamma_2 + 1} \tag{15}$$

where $L_1,\cdots,L_5$ are constant (frequency-dependent) system parameters, which can be expressed in terms of S_{kl}, Γ_1, Γ_3, Γ_4, and e_3. For more details, see [3]. Equation (15) is basic for the transmission measurements described in the companion paper [1].

V. Calibration of the System for Reflection Coefficient Measurements

For calibrating the reflection coefficient measurement setup, the following four reference terminations are used in seven different ways (see Fig. 2 and Table I):

A	adjustable short,	$\Gamma_A = \Gamma e^{j\alpha}$	$(1-\lvert\Gamma\rvert \ll 1)$
B	moveable short similar to A,	$\Gamma_B = \Gamma' e^{j\beta}$	$(1-\lvert\Gamma'\rvert \ll 1)$
C	matched load,	$\Gamma_C = \varepsilon$	$(\lvert\varepsilon\rvert \ll 1)$
D	polished metal plate,	$\Gamma_D = e^{j\pi}$	

It is necessary to know the exact reflection coefficient for only one of the reference objects. In the calibration procedure just presented, Γ_D (polished metal plate) is assumed to be the known coefficient.

Several ways of calibrating the system can be devised. The procedure described in this paragraph is believed to yield most accurate results. It involves moving the short-circuit plane of reference A over one wavelength. The detected signal or its inverse then varies according to certain trigonometric expressions $F(\alpha)$ in each of the procedures I–III (Table I) and IV–VI (Table IV below). The

TABLE I

Ia.	A attached to port 2, C attached to port 3.
Ib.	A attached to port 3, C attached to port 2.
IIab.	A attached to port 2, B attached to port 3.
IIIa.	A attached to port 2, D attached to port 3.
IIIb.	A attached to port 3, D attached to port 2.
IIIc.	B attached to port 2, D attached to port 3.
IIId.	B attached to port 3, D attached to port 2.

variable phase α in these expressions describes the short-circuit plane location in radians. The expressions $F(\alpha)$ are of three types:

Procedures II, IIIa-d, V and VI:

$${}^1F(\alpha) = {}^1K\cdot\frac{\lvert 1+{}^1a\cdot e^{j\alpha}\rvert^2}{\lvert 1+{}^1b\cdot e^{j\alpha}\rvert^2}\left(=\lvert b_4\rvert^2\right) \tag{16a}$$

Procedures Ia and Ib:

$${}^2F(\alpha) = {}^2K\cdot\frac{\lvert 1+{}^2a\cdot e^{j\alpha}\rvert^2}{\lvert 1+{}^2b\cdot e^{-j\alpha}\rvert^2}\left(=\frac{1}{\lvert b_4\rvert^2}\right) \tag{16b}$$

Procedure IV:

$${}^3F(\alpha) = {}^3K\cdot\frac{\lvert 1+{}^3a\cdot e^{-j\alpha}\rvert^2}{\lvert 1+{}^3b\cdot e^{j\alpha}\rvert^2}\left(=\lvert b_4\rvert^2\right). \tag{16c}$$

K is a real parameter while a and b are complex parameters. (Procedures IV–VI noted in (16a) and (16c) are related to transmission measurements described below.)

In each of the calibration procedures I–VI, five quantities $E_0,\cdots,E_4$ are measured. We will show that between these quantities and the parameters ${}^kK$, ${}^ka$, and ${}^kb$ in (16) certain simple relations exist. The quantities $E_0,\cdots,E_4$ are obtained by integrating the function $F(\alpha)$ over one wavelength, viz.,

$$C_0 = E_0 = \frac{1}{2\pi}\int_0^{2\pi} F(\alpha)\,d\alpha \tag{17a}$$

$$C_1 = E_2 - j\cdot E_1 = \frac{1}{2\pi}\int_0^{2\pi} F(\alpha)\cdot e^{-j\alpha}\,d\alpha \tag{17b}$$

$$C_2 = E_4 - j\cdot E_3 = \frac{1}{2\pi}\int_0^{2\pi} F(\alpha)\cdot e^{-2j\alpha}\,d\alpha. \tag{17c}$$

The computer evaluates these integrals by a summation of the detected power values for a number of short-circuit plane positions, distributed over one wavelength. The number of steps N and the steplength Δl are chosen with respect to the desired measurement accuracy.

Inserting

$$F(\alpha) = K\cdot\frac{\lvert 1+a\cdot e^{j\alpha}\rvert^2}{\lvert 1+b\cdot e^{j\alpha}\rvert^2}$$

into (17) leads, for procedures I–VI, to

$$C_0=\frac{K}{1-|b|^2}\cdot\left(1+|a|^2-ab\quad y-\bar{a}b\right) \tag{18a}$$

$$C_1=\frac{K}{1-|b|^2}\left(a-\left(1+|a|^2\right)\cdot b+\bar{a}b^2\right) \tag{18b}$$

$$C_2=\frac{K}{1-|b|^2}\cdot\left(-ab+\left(1+|a|^2\right)\cdot b^2-\bar{a}\cdot b^3\right) \tag{18c}$$

From the relations in (18), K (real) and a and b (complex) can be determined analytically as follows:

$$b=-\frac{C_2}{C_1}$$

$$a=\bar{Q}\cdot\left(1\underset{(+)}{-}\sqrt{1-4/|Q|^2}\right)/2$$

$$K=\frac{2\cdot|C_1+C_0 b|}{|Q|\cdot\left(1\underset{(+)}{-}\sqrt{1-4/|Q|^2}\right)} \tag{19}$$

where

$$Q=\frac{C_0\left(1+|b|^2\right)+2\cdot\mathrm{Re}\left(C_1\cdot\bar{b}\right)}{C_1+C_0\cdot b}.$$

Notice that two solutions exist for a and K above. Those for $|a|>1$ are indicated with a "+" sign in front of the square root. The parameters ${}^kK$, ${}^ka$, and ${}^kb$ of the ${}^kF(\alpha)$ functions in (16) are related to K, a, and b in (19) as follows:

$$\left.\begin{matrix} {}^1K=K & {}^1a=a & {}^1b=b \\ {}^2K=K & {}^2a=a & {}^2b=\bar{b} \\ {}^3K=K & {}^3a=\bar{a} & {}^3b=b \end{matrix}\right\}. \tag{20}$$

The parameter indexes above are used in Tables II, III, and V. If dissipation is nonnegligable, a recursive calculation procedure can be used for more accurate values of $E_0,\cdots,E_4$ [4]. The calculation of $|a|$ using (19) leads to results which are sensitive for measurement errors if $|a|\approx 1$. For this case, a separate measurement of $|a|$ can be recommended [4].

In order to evaluate the seven system parameters $K_1,\cdots,K_7$ from a^k, b^k, and K^k, as obtained from (19) and (20), we have to identify the expressions (16a–c) with the actual equations. The latter are obtained by recognizing that $|b_4|^2$ can be obtained from (11) using the proper calibration terminations for Γ_2 and Γ_3 according to Table I. The result is shown in Table II.

TABLE II

$${}^2K_{Ia}=\frac{|1+K_7\cdot\varepsilon|^2}{|K_2|^2\cdot|\Gamma|^2\cdot\left|1+\frac{K_1}{K_2}\cdot\varepsilon\right|^2}$$

$${}^2a_{Ia}=\frac{\Gamma K_6+\Gamma K_5\cdot\varepsilon}{1+K_7\cdot\varepsilon} \qquad {}^2b_{Ia}=\frac{\frac{K_4}{\Gamma K_2}+\frac{K_3}{\Gamma K_2}\cdot\varepsilon}{1+\frac{K_1}{K_2}\cdot\varepsilon}$$

$${}^2a_{Ib}=\frac{\Gamma K_7+\Gamma K_5\cdot\varepsilon}{1+K_6\cdot\varepsilon} \qquad {}^2b_{Ib}=\frac{\frac{K_4}{\Gamma K_3}+\frac{K_2}{\Gamma K_3}\cdot\varepsilon}{1+\frac{K_1}{K_3}\cdot\varepsilon}$$

$${}^3a_{IIa}\cdot{}^3a_{IIb}=-\left(\frac{K_3}{K_2}\right)^2\cdot\left(\frac{\Gamma'}{\Gamma}\right)^2\cdot\frac{1-\left(\frac{K_4}{K_3\cdot\Gamma'}\right)^2}{1-\left(\frac{K_1\cdot\Gamma'}{K_2}\right)^2}$$

$${}^1a^{+}_{IIIa}\cdot{}^1a_{IIIb}=\Gamma^2\cdot F \qquad {}^1a^{+}_{IIIc}\cdot{}^1a_{IIId}=\Gamma'^2\cdot F$$

where

$$F=\frac{\left(1-\left(\frac{K_1}{K_2}+\frac{K_1}{K_3}\right)+\frac{K_1^2}{K_2\cdot K_3}\right)}{\left(1-\left(\frac{K_4}{K_2}+\frac{K_4}{K_3}\right)+\frac{K_4^2}{K_2\cdot K_3}\right)}$$

$${}^1a^{+}_{IIIa}=-\left(\frac{K_2}{K_3}\right)\cdot\Gamma\cdot\frac{\left(1-\frac{K_1}{K_2}\right)}{\left(1-\frac{K_4}{K_3}\right)} \qquad {}^1b_{IIIa}=\frac{K_6\Gamma-K_5\Gamma}{1-K_7}$$

*) ${}^1a^{+}$ indicates solutions of (19)–(20) for which $|a|>1$.

In each of these expressions, a dominating term can be found. In an idealized system (involving an ideal magic T), all system parameters except K_2 and K_3 are equal to zero. However, for the system in use, it is quite sufficient if the following conditions are fulfilled:

$$\left|\frac{K_1}{K_2}\right|,\left|\frac{K_1}{K_3}\right|,\left|\frac{K_4}{K_2}\right|,\left|\frac{K_4}{K_3}\right|\le q\ll 1. \tag{21}$$

Most magic T's can easily satisfy the requirements (typically $0.01<q<0.1$). If (21) is satisfied, fast convergence in the calculation procedures (Table III) are obtained. The exact upper limit of q in (21) for obtaining convergence has not yet been investigated.

By using the iterative calculation procedures suggested below, results are obtained with an accuracy primarily limited only by the linearity of the detector. In Table III, the iterative procedure is described. Only a few iterations are necessary for a high degree of accuracy, and microwave components with moderate quality specifications can be used with maintained accuracy of reflection measurements. Only the magnitude of $|K_2|$ is influenced by the input power level during the calibration procedure Ia. All other expressions are independent of the power level used.

VI. Calibration of the System for Transmission Measurements

In order to determine the five system parameters $L_1,\cdots,L_5$ of (15), the calibration procedures in Table IV (see also Fig. 5) have to be performed. A similar procedure as described above leads to the relations shown in Table V (for more details, see [4]).

From the equations shown in Table V, one observes that exact values of all L_K parameters needed are obtained without any iterative calculation procedure.

TABLE III

$$(\Gamma)_o = \pm\sqrt{{}^1a_{IIIa}\cdot{}^1a_{IIIb}} \quad {}^{*)} \qquad (\Gamma)_{n+1} = \pm\sqrt{{}^1a_{IIIa}\cdot{}^1a_{IIIb}}\Big/\sqrt{F_n} \quad {}^{*)}$$

$$(\Gamma')_o = \pm\sqrt{{}^1a_{IIIc}\cdot{}^1a_{IIId}} \quad {}^{*)} \qquad (\Gamma')_{n+1} = \pm\sqrt{{}^1a_{IIIc}\cdot{}^1a_{IIId}}\Big/\sqrt{F_n} \quad {}^{*)}$$

where

$$F_n = \frac{1-\left(\frac{K_1}{K_2}\right)_n\cdot\left[\left(\frac{K_3}{K_2}\right)_n^{-1}+1\right]+\left(\frac{K_1}{K_2}\right)_n^2\cdot\left(\frac{K_3}{K_2}\right)_n^{-1}}{1-\left(\frac{K_4}{K_2}\right)_n\cdot\left[\left(\frac{K_3}{K_2}\right)_n^{-1}+1\right]+\left(\frac{K_4}{K_2}\right)_n^2\cdot\left(\frac{K_3}{K_2}\right)_n^{-1}}$$

$$\left(\frac{K_3}{K_2}\right)_0 = \pm\sqrt{-{}^3a_{IIa}\cdot{}^3a_{IIb}}\cdot\frac{(\Gamma)_0}{(\Gamma')_0} \quad {}^{**)} \qquad \left(\frac{K_3}{K_2}\right)_{n+1} = \pm\sqrt{-{}^1a_{IIa}\cdot{}^1a_{IIb}}\cdot\frac{(\Gamma)_{n+1}}{(\Gamma')_{n+1}}\,\frac{1-\left(\frac{K_1}{K_2}\right)_n^2\cdot(\Gamma')_n^2}{1-\left(\frac{K_4}{K_2}\right)_n^2\cdot\left(\frac{K_3}{K_2}\right)_n^2\cdot(\Gamma')_n^{-2}} \quad {}^{**)}$$

$$(K_6)_0 = {}^2a_{Ia}\cdot(\Gamma)_0^{-1} \qquad (K_6)_{n+1} = {}^2a_{Ia}\cdot(1+(K_7)_n\cdot\varepsilon_n)\cdot(\Gamma)_{n+1}^{-1}-(K_5)_n\cdot\varepsilon_n$$

$$(K_7)_0 = {}^2a_{Ib}\cdot(\Gamma)_0^{-1} \qquad (K_7)_{n+1} = {}^2a_{Ib}\cdot(1+(K_6)_n\cdot\varepsilon_n)\cdot(\Gamma)_{n+1}^{-1}-(K_5)_n\cdot\varepsilon_n$$

$$\left(\frac{K_4}{K_2}\right)_0 = \frac{{}^2b_{Ia}-\left(\frac{K_3}{K_2}\right)_0^2\cdot{}^2b_{Ib}}{1-\left(\frac{K_3}{K_2}\right)_0}\cdot(\Gamma)_0 \qquad \left(\frac{K_4}{K_2}\right)_{n+1} = {}^2b_{Ia}\cdot\left[1+\left(\frac{K_1}{K_2}\right)_n\cdot(\varepsilon)_n\right]\cdot(\Gamma)_{n+1}-\left(\frac{K_3}{K_2}\right)_{n+1}\cdot(\varepsilon)_n$$

$$(\varepsilon)_0 = \frac{{}^2b_{Ia}-\left(\frac{K_3}{K_2}\right)_0\cdot{}^2b_{Ib}}{\left(\frac{K_3}{K_2}\right)_0-1}\cdot(\Gamma)_0 \qquad (\varepsilon)_{n+1} = \frac{{}^2b_{Ia}-\left(\frac{K_4}{K_2}\right)_{n+1}\cdot(\Gamma)_{n+1}^{-1}}{\left(\frac{K_3}{K_2}\right)_{n+1}\cdot(\Gamma)_{n+1}^{-1}-{}^2b_{Ia}\cdot\left(\frac{K_1}{K_2}\right)_n}$$

$$\left(\frac{K_1}{K_2}\right)_0 = 1+\frac{1}{(\Gamma)_0}\cdot\left[\left(\frac{K_3}{K_2}\right)_0-\left(\frac{K_4}{K_2}\right)_0\right]\cdot{}^1a_{IIIa}^{+} \qquad \left(\frac{K_1}{K_2}\right)_{n+1} = 1+\left[\left(\frac{K_3}{K_2}\right)_{n+1}-\left(\frac{K_4}{K_2}\right)_{n+1}\right]\cdot(\Gamma)_{n+1}^{-1}\cdot{}^1a_{IIIa}^{+}$$

$$(K_5)_0 = \left[{}^2a_{Ia}-b_{IIIa}\cdot(1-(K_7)_0)\right]\cdot(\Gamma)_0^{-1} \qquad (K_5)_{n+1} = (K_6)_{n+1}-{}^1b_{IIIa}\cdot(1-(K_7)_{n+1})\cdot(\Gamma)_{n+1}^{-1}$$

$$|K_2|_0 = |\Gamma|_0^{-1}\cdot\frac{1}{\sqrt{{}^2K_{Ia}}} \qquad |K_2|_{n+1} = |\Gamma_{n+1}|^{-1}\cdot\frac{1}{\sqrt{{}^2K_{Ia}}}\cdot\frac{|1+(K_7)_{n+1}\cdot\varepsilon_{n+1}|}{\left|1+\left(\frac{K_1}{K_2}\right)_{n+1}\cdot\varepsilon_{n+1}\right|}$$

*) That sign is selected which gives an argument for (Γ) most like the argument of ${}^1a_{IIIa}$ and an argument of (Γ') most like the argument of ${}^1a_{IIIc}$.

**) That sign is selected which gives the argument for (K_3/K_2) most like π.

TABLE IV

IV.	A attached to port 2, matched load attached to ports 5, 6 and 8, source fed to port 7
V.	A attached to port 2, matched loads attached to port 6, 7 and 8, source fed to port 5
VI.	A attached to port 2, waveguide attached to ports 6 and 7, source fed to port 5.

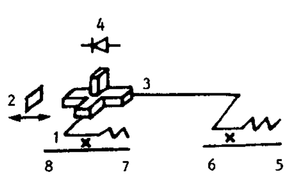

Fig. 5. Basic arrangement for transmission parameter measurements.

TABLE V

$$\frac{L_1}{L_4} = \frac{({}^1a_{VI}-{}^1a_V)/\Gamma}{1-{}^1a_{VI}\cdot{}^3a_{IV}}$$

$$\frac{L_2}{L_4} = {}^3a_{IV}\cdot\frac{L_1}{L_4}\cdot\Gamma$$

$$\frac{L_3}{L_4} = {}^1a_V/\Gamma$$

$$L_5 = {}^3b_{IV}/\Gamma$$

$$|L_4| = \sqrt{{}^1K_V}$$

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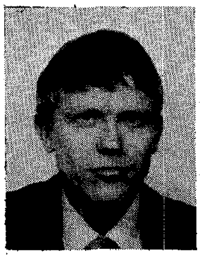

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